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Number theory

Fall Semester 2013

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Quizz #1

Due Friday 4 october in recitation.

Problems:

1. Prove that for every integer n, $n^2 - n$ is divisible by 2.
2. Prove that there is no prime integer $p > 2$ such that $p^2 + 1$ is also prime.
3. Find all the integers $1 \leq x \leq 20$ such that $gcd(x, 15) = 5$.
4. Find all the integer solutions of the equation $8m + 6n = 2$.
5. Deduce all the congruence classes mod 6 solutions of $8x \equiv 2 \; mod \; 6$.

Solution:

1. Notice that $n^2 - n = n(n-1)$. If n is odd, then $n - 1$ is even and thus divisible by 2, so 2 divides $n^2 - n$. If n is even then divisible by 2 and $n^2 - n$ is divisible by 2.
2. If p is prime, p is odd, then p^2 is also odd and $p^2 + 1$ is even, so can not be a prime.
3. $gcd(x, 15) = 5$, so 5 divides x then 5, 10, 15, 20 but 3 does not divides x otherwise $gcd(x, 15) = 15$, so $x = 5, 10, 20$.
4. $gcd(8, 6) = 2|2$ so there is solutions. Clearly $m = 1$ and $n = -1$ is a solution. Let (m, n) another solution. Then $8m + 6n = 8 - 6 = 2$ and $4(m-1) = 3(1-n)$, since $gcd(4, 3) = 1$ then $4|(1-n)$ and there is a integer k such that $1 - n = 4k$ but then $m - 1 = 3k$. So, a general solution is $n = -4k - 1$ and $m = 3k + 1$.
5. We notice that $8x \equiv 2 \; mod \; 6$ is equivalent to the existence of an integer n such that $8x + 6n = 2$. By the previous question, the general solution x is of the form $x = 3k + 1$ which gives two congruence classes mod 6, $x \equiv 1 \; mod \; 6$ and $x \equiv 4 \; mod \; 6$.